



# Distinguishing response behaviors within cumulative noise metrics for Quiet Supersonic Flights 2018 data

Aaron B. Vaughn and Andrew W. Christian  
*NASA Langley Research Center*

March 21, 2023  
Acoustics Technical Working Group Meeting

# Introduction

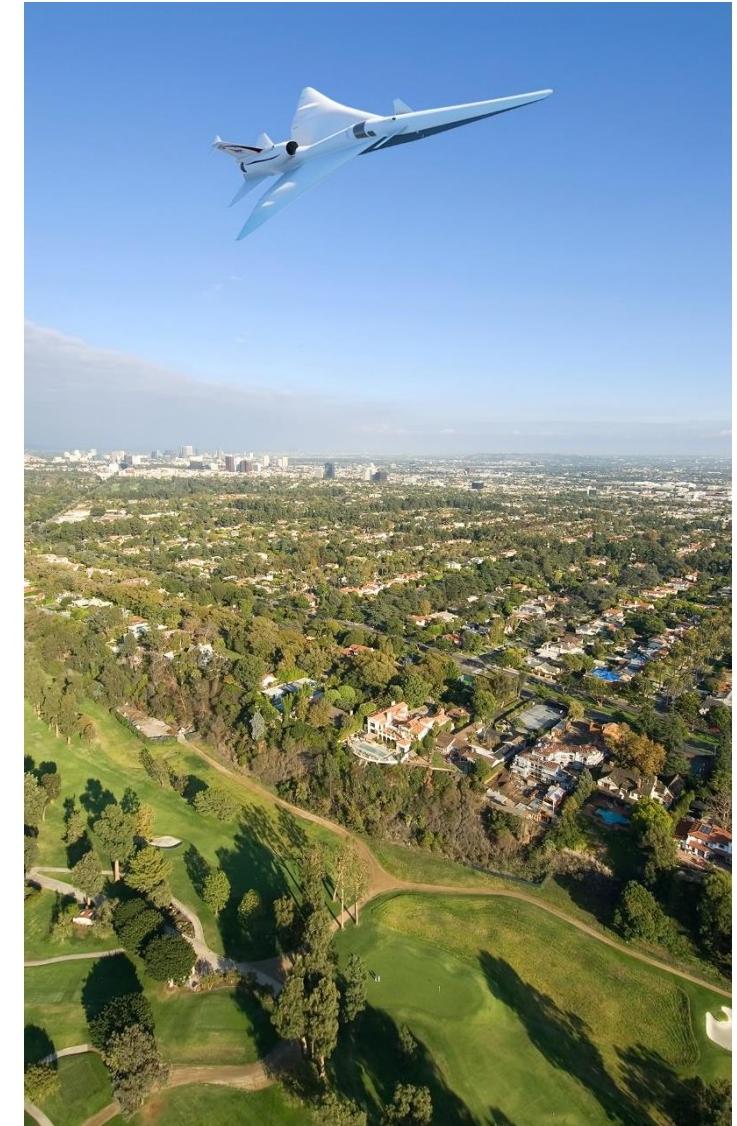
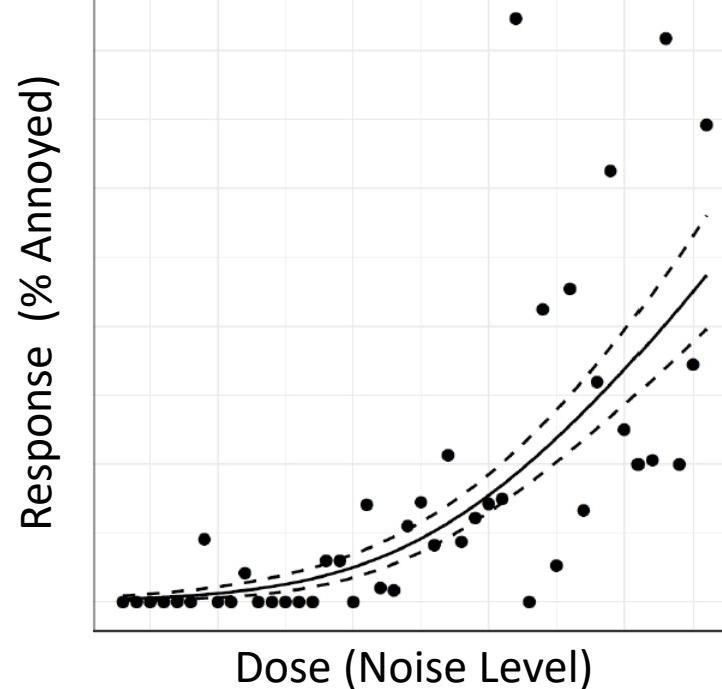
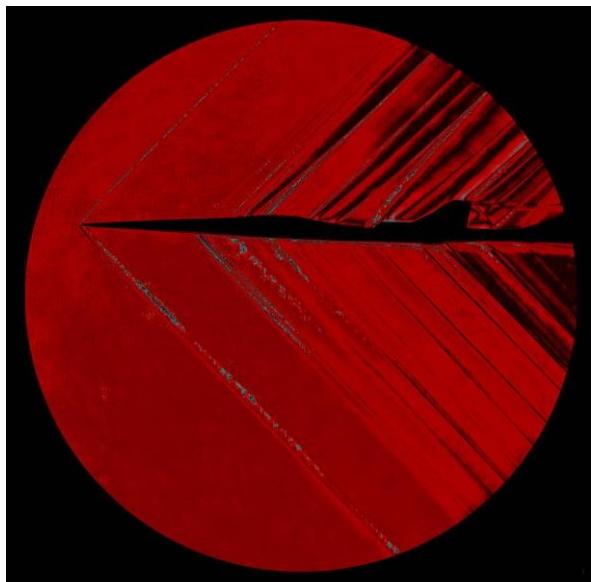
---



## ➤ Upcoming X-59 community tests

- Ban on overland commercial supersonic flight
- Shaped-boom technology → Quiet supersonic flight
- Noise dose and annoyance response data to inform regulators

## ➤ How do people respond to multiple supersonic overflights in a day?



# Dose-Response Data



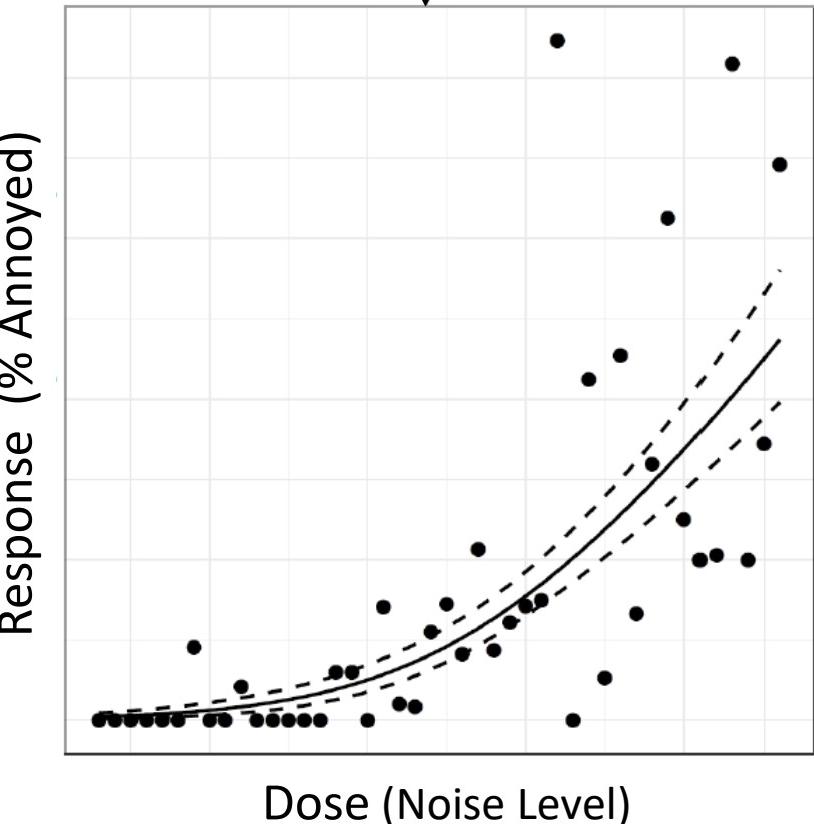
How much did the sonic boom bother, disturb, or annoy you?

1      2      3      4      5

Convert to binary  
“Highly Annoyed” (HA) response

Participant ID	Event Number	Dose (Noise Level)	Annoyance Response
001	1	70	0
001	2	81	1
001	3	74	0
002	1	72	0
002	2	78	0

Fit Statistical Model



# Single-Event and Cumulative Dose-Response Data



➤ **Longitudinal (panel) Study Design:** each participant responds to multiple events

- Single Event (SE)

- Responses to individual flyovers
- Perceived Level (PL) [dB]

- Cumulative

- Response from end-of-day survey
- Perceived Day-Night Level (PLDNL) [dB]
  - Summation of single events ( $SE_i$ ) in one day:

$$L_{dn} = 10 \log_{10} \left( \sum_i 10^{SE_i/10} \right) - 49.4$$

Participant ID	Event Number	Dose (PL)	Annoyance Response
001	1	70	0
001	2	81	1
001	3	74	0
002	1	72	0
002	2	78	0

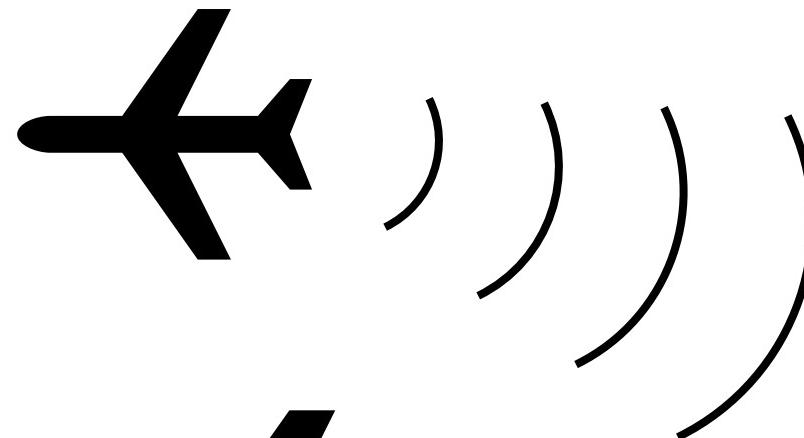
Participant ID	Day	Dose (PLDNL)	Annoyance Response
001	1	32.7	1
002	1	29.6	0

# Potential Predictors of Cumulative Annoyance



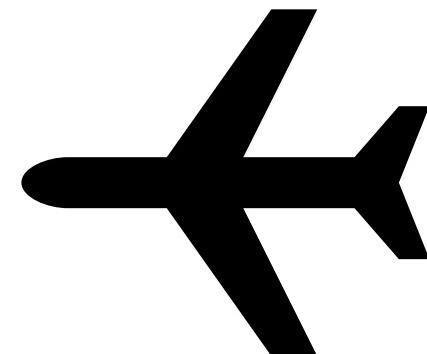
## ➤ Day-Night Level (DNL)

- Equal Energy Hypothesis
  - Level
  - Duration
  - Number of events



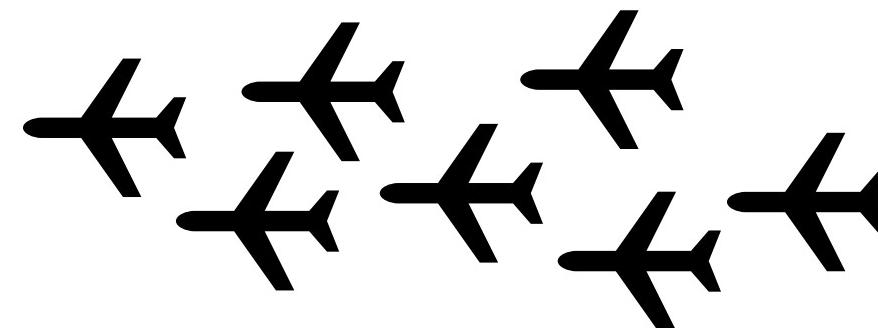
## ➤ Loudest single event

- Peak-End Rule



## ➤ Number of events

- Noise and Numbers
  - E.g., Fields 1983



# Introducing $\beth$ (“Bet”)



- Reformulating DNL equation:

$$L_{dn,\beth} = 20 \log_{10} \left[ \left( \sum_{i=1}^N \left( 10^{\frac{SE_i}{20}} \right)^{\beth^{-1}} \right)^{\beth} \right] - 49.4$$

- Based on generalized vector norms ( $\beth = p^{-1}$ )
- $\beth$  (“bet”) is bounded from 0 to 1
- Interpretation given in table:

$\beth$	$p$ -norm	$L_{dn,\beth}$ returns:	Importance given to:
0	$\infty$	Maximum of $SE_i$ vector	Loudest Single Event
0.5	2	Original DNL	Equal Energy
1	1	Coherent summation	Number of Events

# ג Example



- Consider 3 combinations of SE:

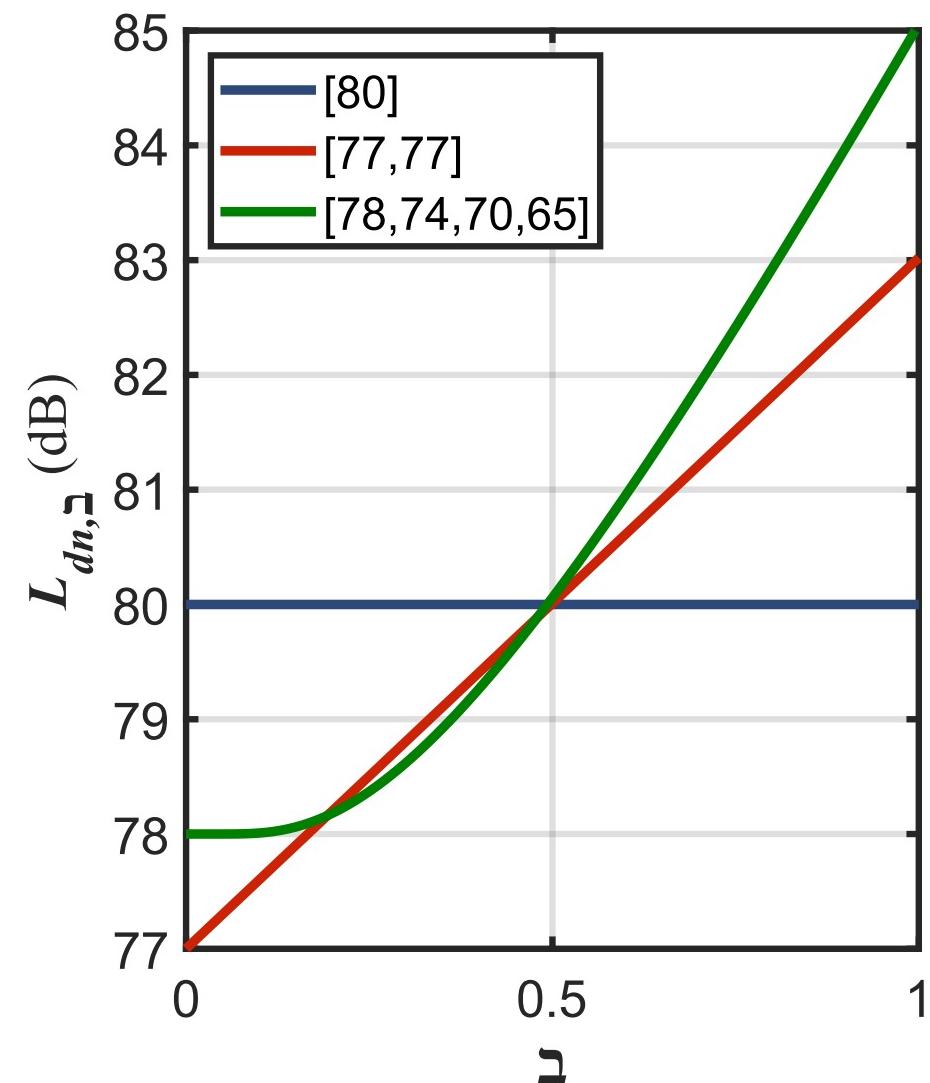
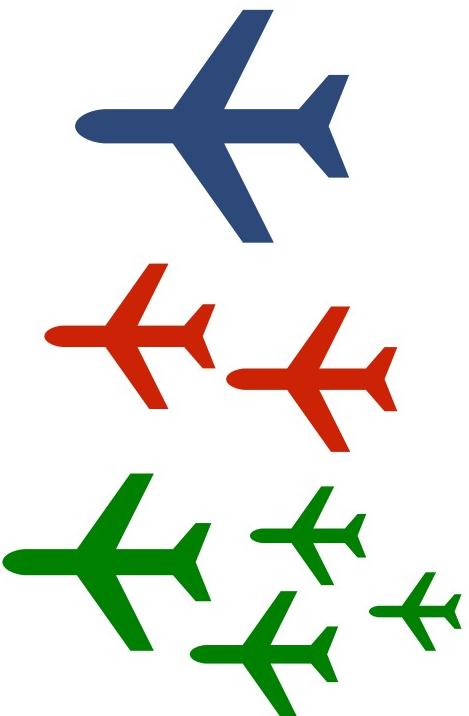
- $SE_i = [80]$  (dB)
- $SE_i = [77, 77]$  (dB)
- $SE_i = [78, 74, 70, 65]$  (dB)

- Combine using  $L_{dn,\gamma}$ :

$$L_{dn,\gamma} = 20 \log_{10} \left[ \left( \sum_{i=1}^N \left( 10^{\frac{SE_i}{20}} \right)^{\gamma^{-1}} \right) \right] - 49.4$$

(For simplicity, drop 49.4)

ג	$L_{dn,\gamma}$ (dB)	$L_{dn,\gamma}$ (dB)	$L_{dn,\gamma}$ (dB)	Importance given to:
0	80	77	78	Loudest Single Event
0.5	80	80	80	Equal Energy
1	80	83	85	Number of Events



# Analysis Methods

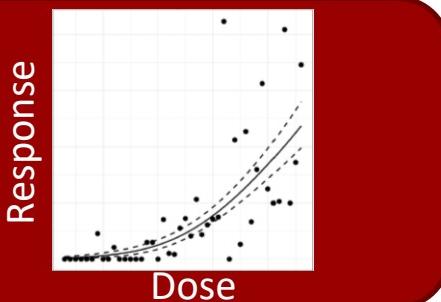


For each  $\gamma$  value from 0 to 1 in small (0.001) increments:

1) Calculate  $L_{dn,\gamma}$  values:

$$L_{dn,\gamma} = 20 \log_{10} \left[ \left( \sum_{i=1}^N \left( 10^{\frac{SE_i}{20}} \right)^{\gamma^{-1}} \right)^\gamma \right] - 49.4$$

2) Fit logistic curve to  $L_{dn,\gamma}$  dose-response data:



3) Evaluate Log Likelihood:

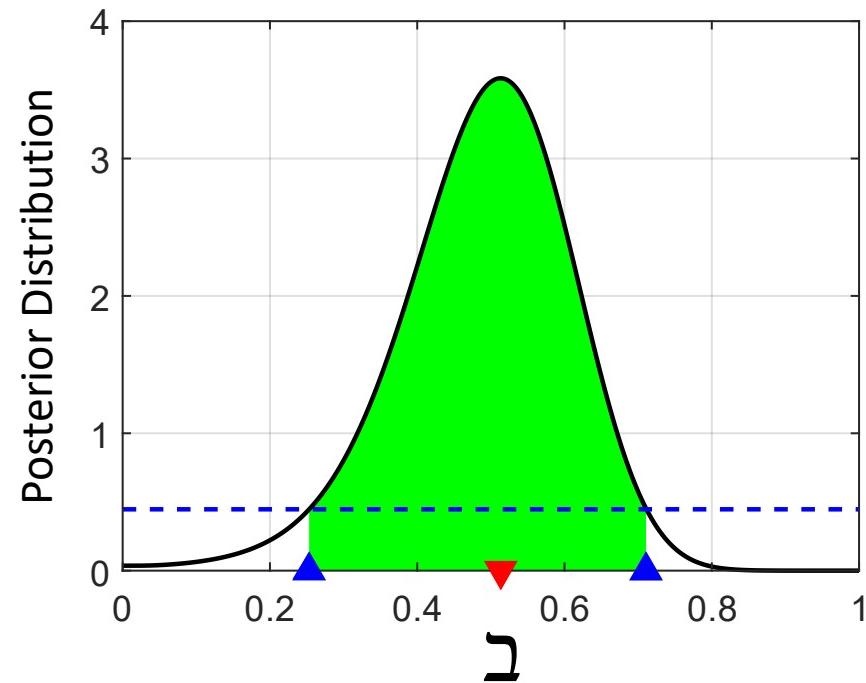
"the likelihood of the data given  $\gamma$ "

$$L_{dn,\gamma}(\text{Data}|\gamma) = \log \left( \prod_{i=1}^n A^{\text{HA}}(x_i) [1 - A^{\text{nHA}}(x_i)] \right)$$

4) Compute Posterior Distribution:  
"the probability of  $\gamma$  given the data"

$$Po(\gamma|\text{Data}) = \frac{L_{dn,\gamma}(\text{Data}|\gamma) \cdot Pr(\gamma)}{P(\text{Data})}$$

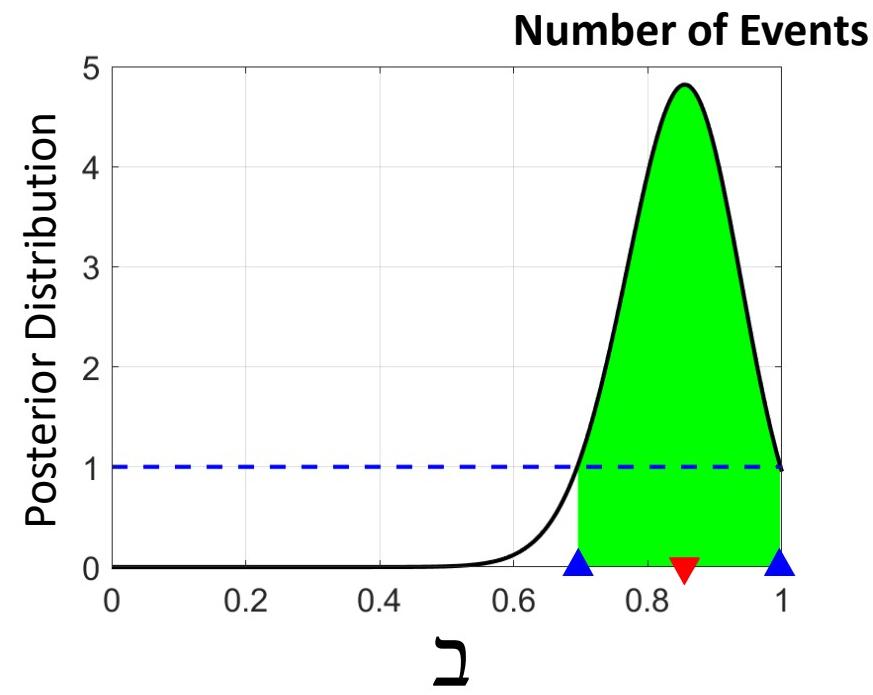
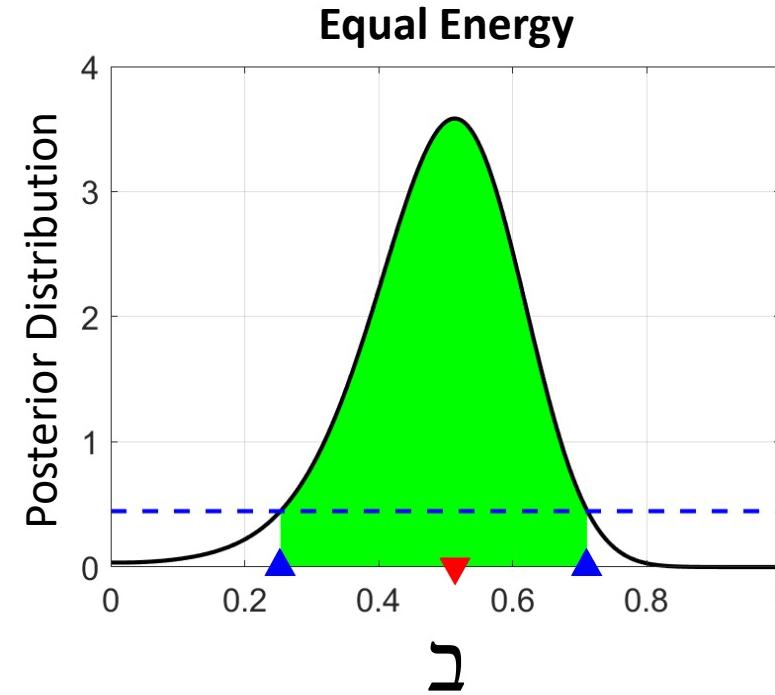
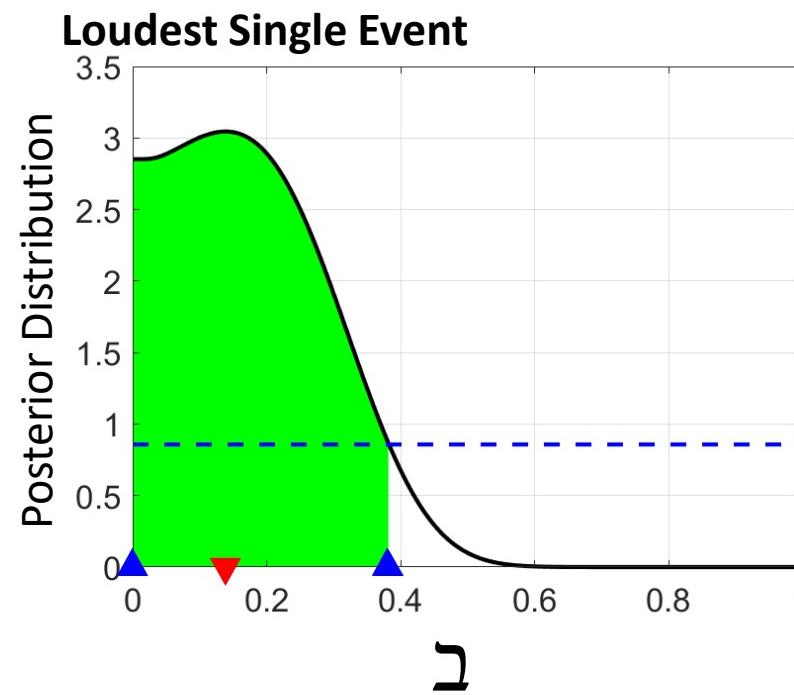
5) Plot  $\gamma$  Posterior Distribution:



# Examples of Potential $\beth$ Results



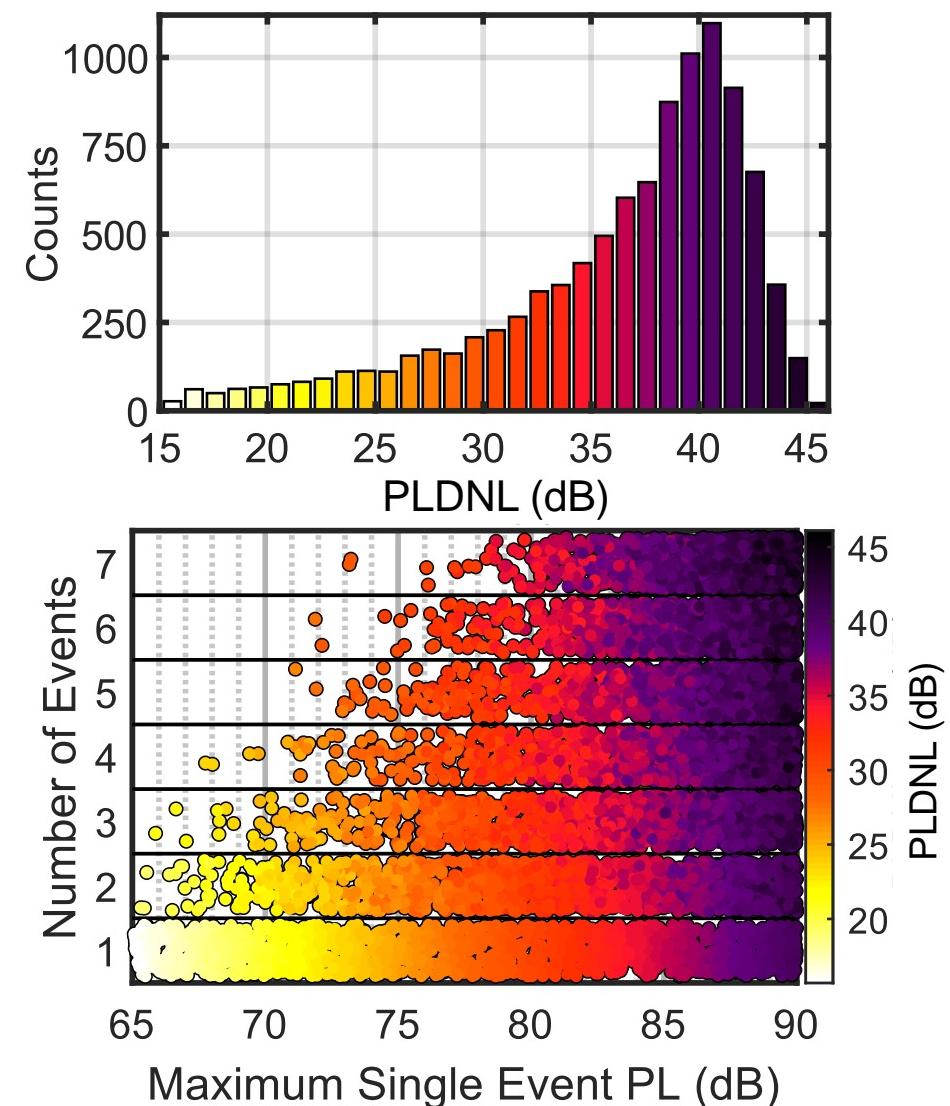
- Plots not from real data, only illustrative of potential results
- Questions to consider when interpreting results:
  - Where is the peak?
  - How narrow/broad is the peak?
  - What  $\beth$  values can be rejected?



# Simulation Dose Data



- **Simulation goals:**
  - Demonstrate  $\Sigma$  analysis
  - Provides context for understanding QSF18 results
- 10,000 cumulative dose-response pairs
- Dose:
  - Number of single events: 1 to 7 from uniform distribution
  - Single-event dose range: 65 to 90 dB from uniform distribution
- Maximum Single Event PL vs Number of Events
  - Top left corner empty
    - Due to logarithmic relationship & uniform distributions
- Need to define responses



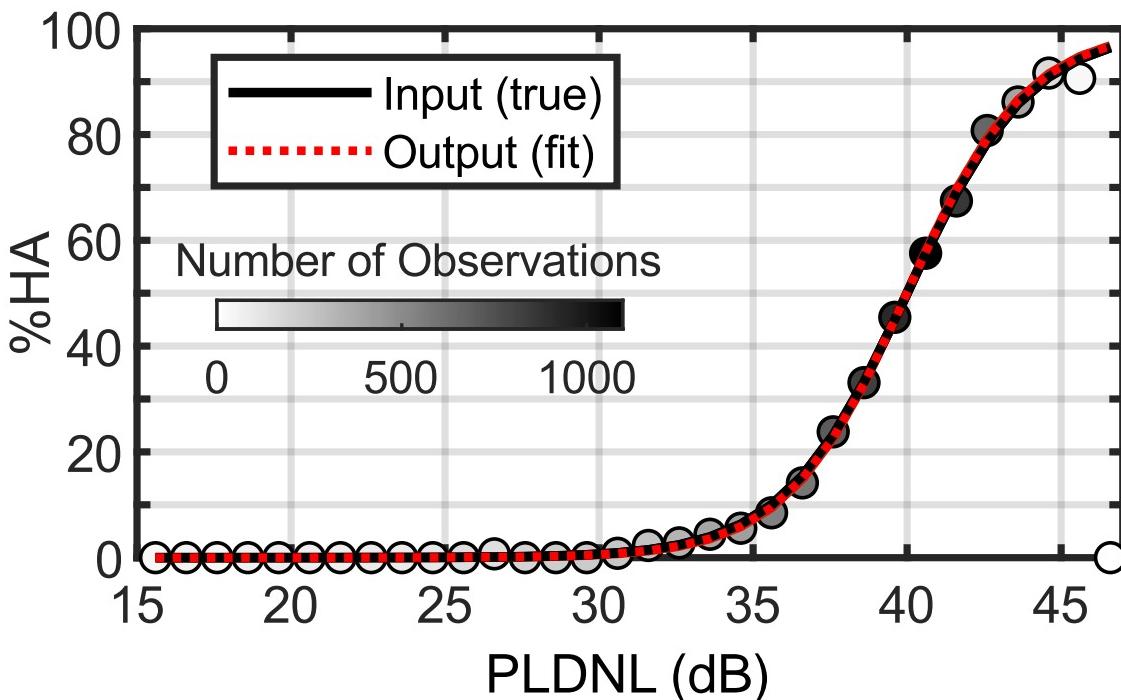
# Simulation Response Data



- Simulations with same doses but differing responses
- Specify  $\gamma = 0.5$  as “true” response

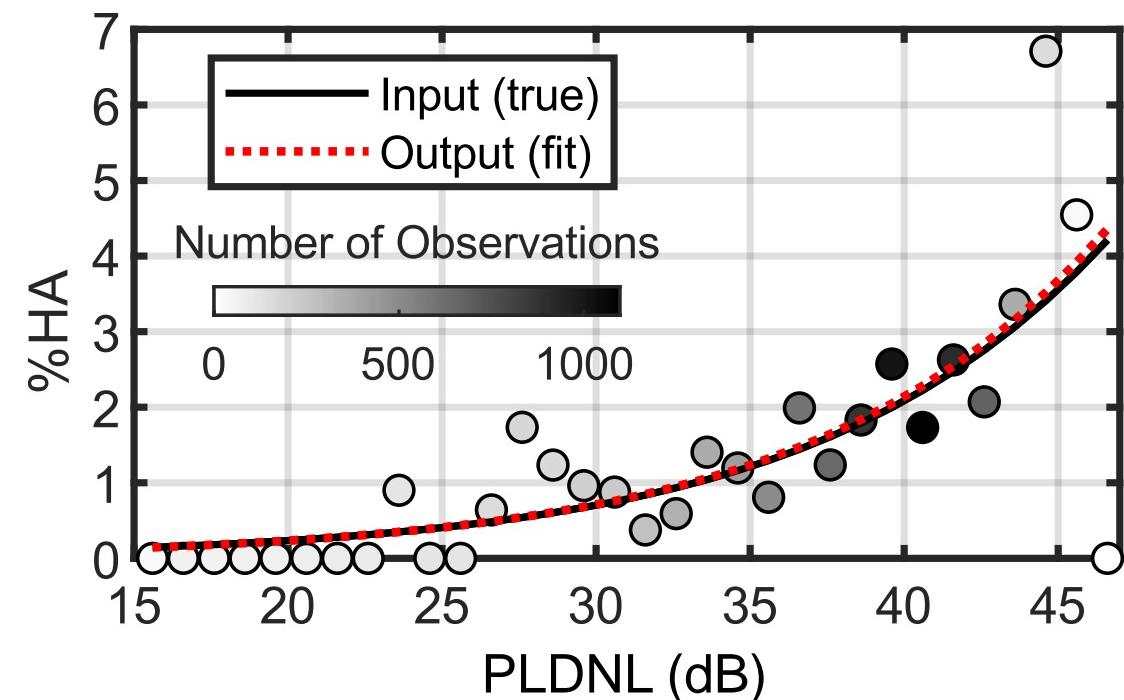
- **Simulation 1:**

- Fully Sampled Dose-Response Curve
- %HA from 0 to almost 100%HA



- **Simulation 2:**

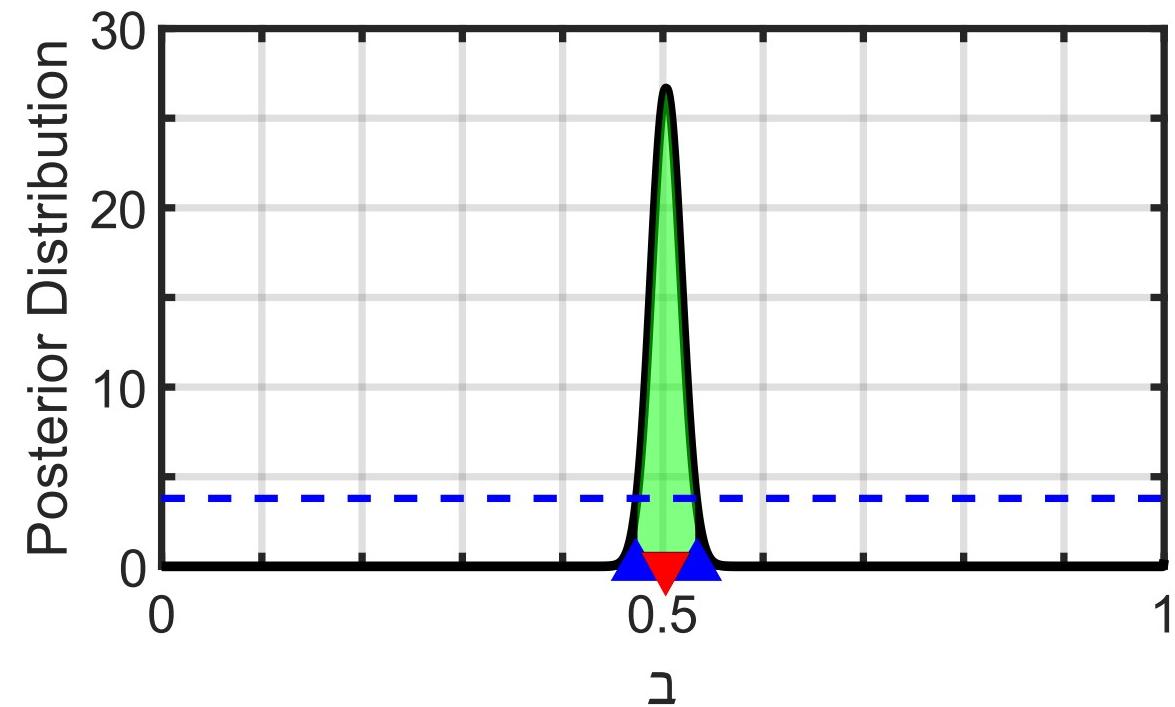
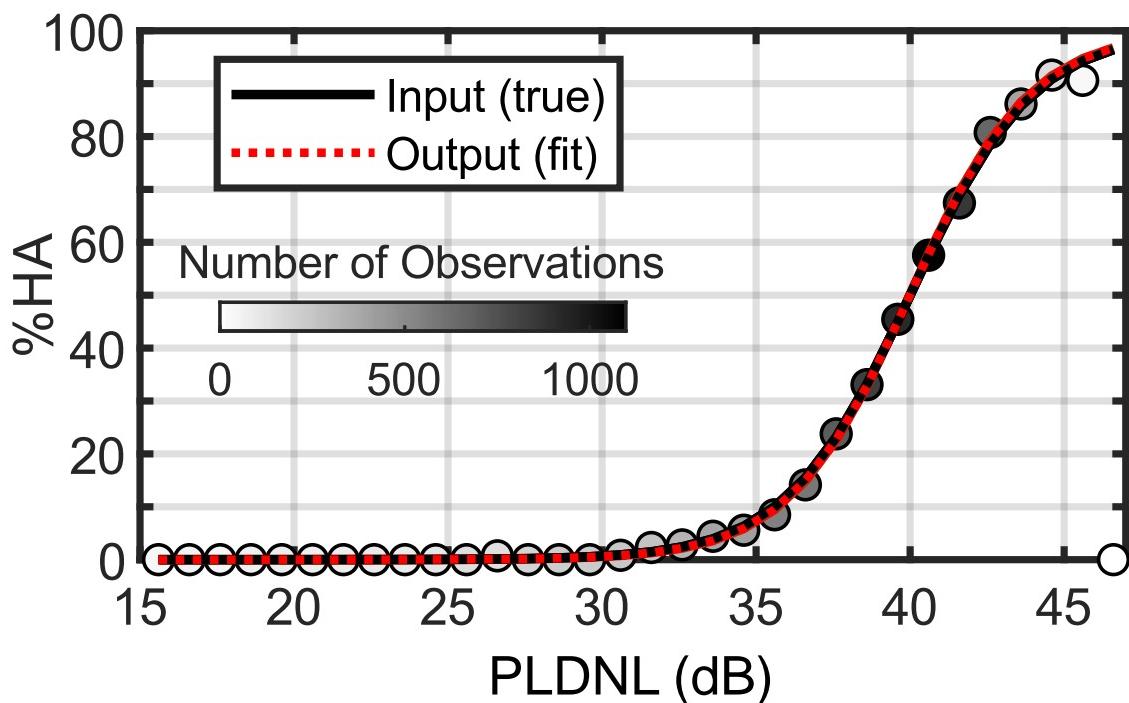
- Poorly Sampled Dose-Response Curve
- %HA from 0 to about 4%HA



# Simulation 1 Results: Fully Sampled Dose-Response Curve



- Where is the peak?
  - How narrow/broad is the peak?
  - What  $\gamma$  values can be rejected?
- $\gamma \approx 0.5$ ; **accurate**
  - Narrow peak; **precise**
  - $\gamma = 0$  &  $1$  can be confidently rejected

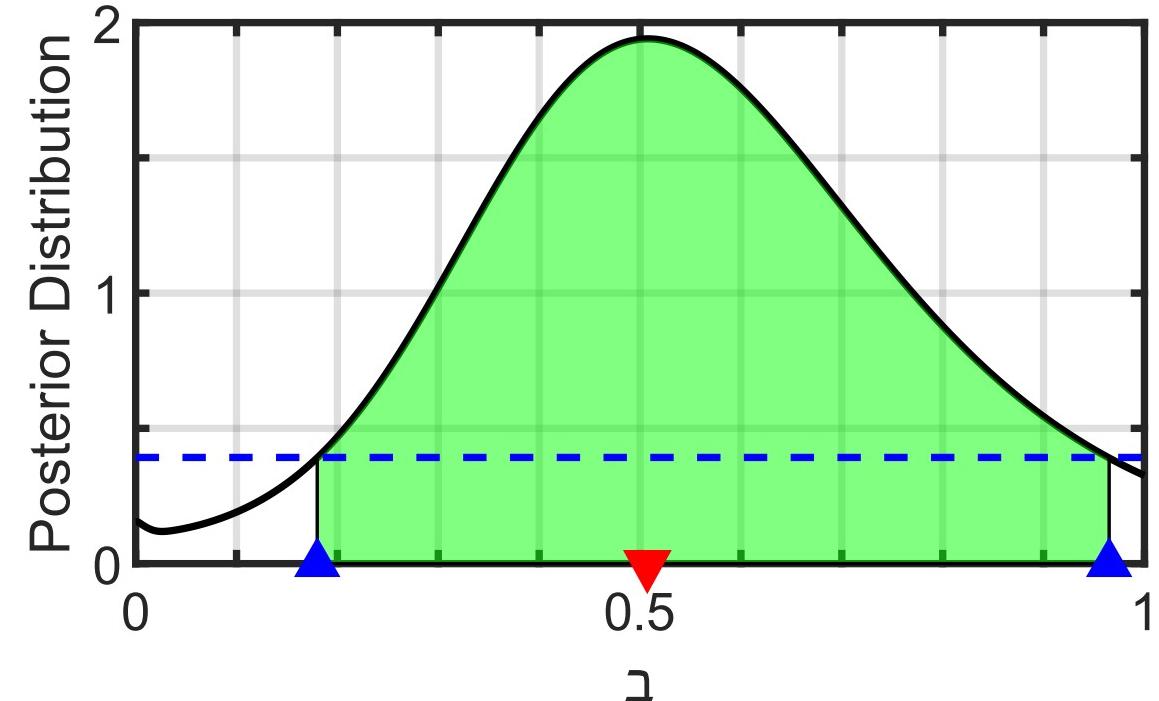
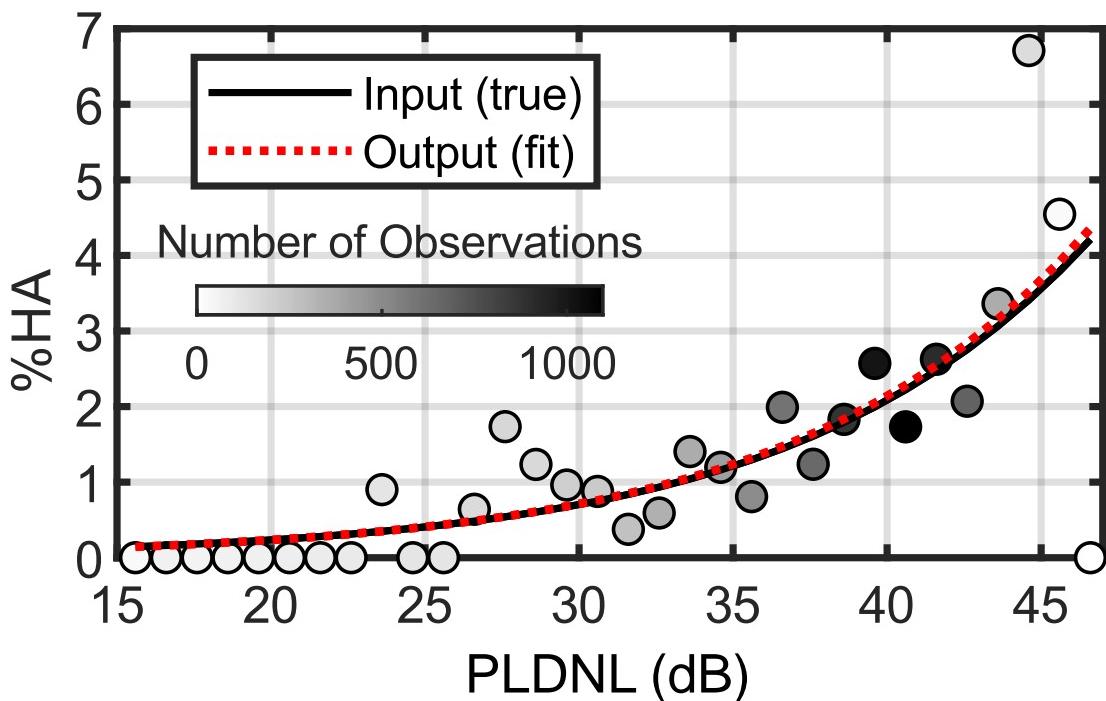


# Simulation 2 Results: Poorly Sampled Dose-Response Curve



- Where is the peak?
- How narrow/broad is the peak?
- What  $\gamma$  values can be rejected?

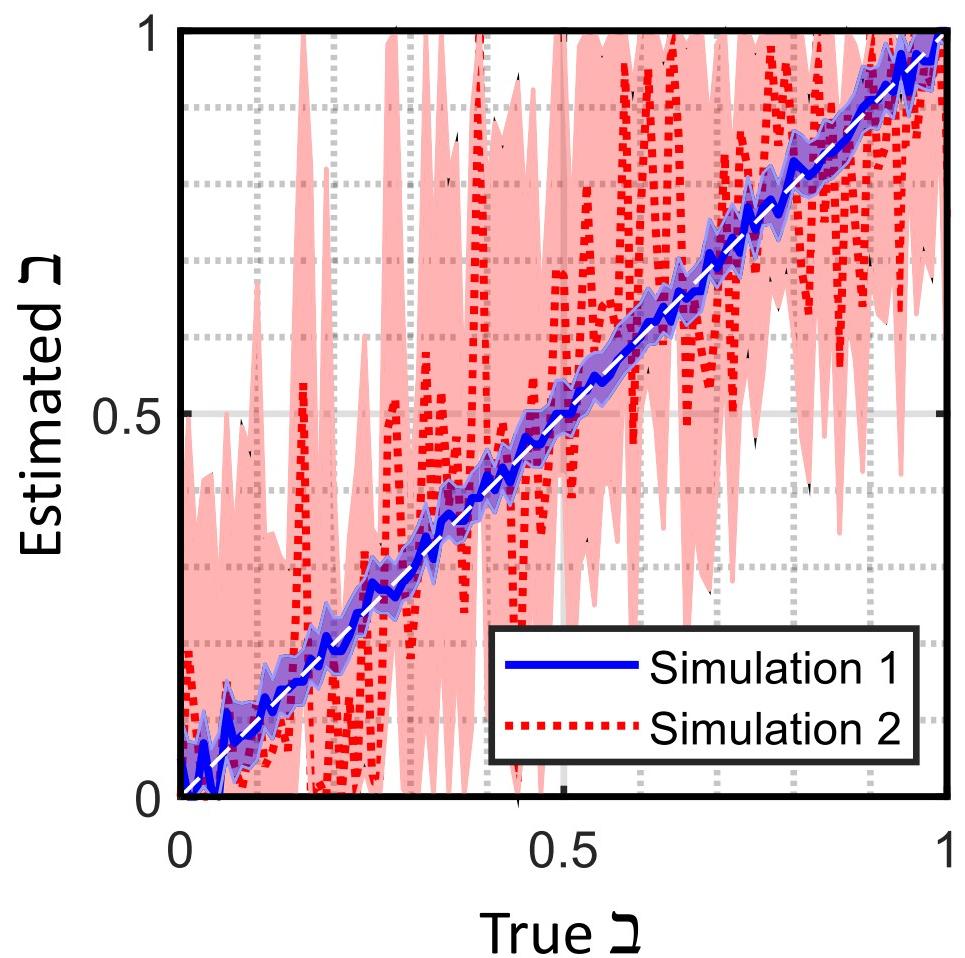
- $\gamma \approx 0.5$ ; somewhat accurate
- Very broad peak; not precise
- $\gamma = 0$  & 1 cannot be confidently rejected



# Simulation Results for Various Input $\gamma$ Values



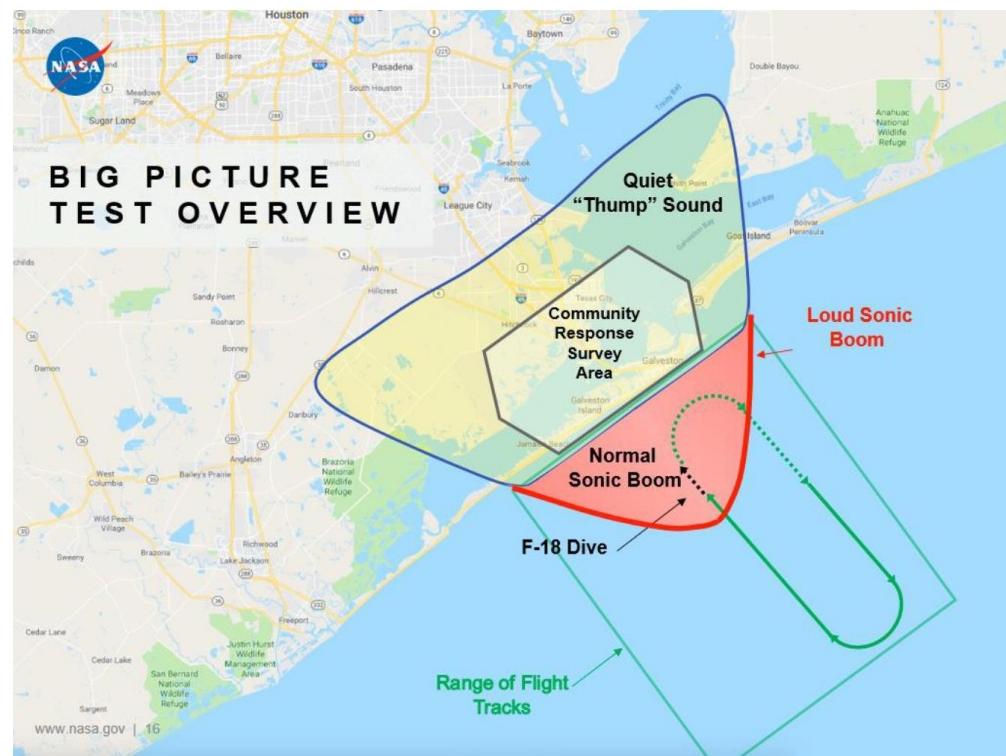
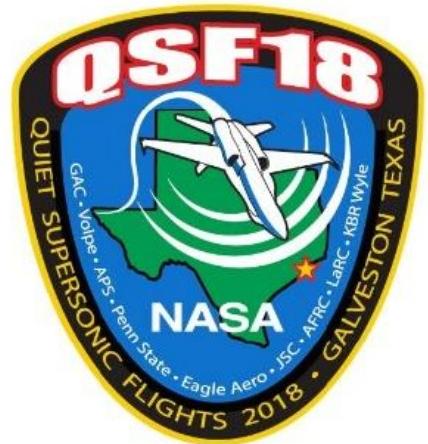
- Previous examples set input  $\gamma$  to 0.5
- Now vary  $\gamma$  from 0 to 1 in 0.01 steps for both cases
  - Simulation 1 is fully sampled dose-response curve
  - Simulation 2 is poorly sampled dose-response curve
- Results:
  - Both simulations are accurate
    - Input  $\gamma$  captured within output  $\gamma$  CI
  - Simulation 1 more precise
  - Simulation 2 rejects fewer  $\gamma$  values



# Quiet Supersonic Flights 2018 (QSF18)



- Galveston, Texas in November 2018
- Low-amplitude sonic booms via F-18 dive maneuver
- 9 Flight days
  - 52 total flyover events
- 385 provided  $\geq 1$  cumulative response
- **1952 total cumulative dose-response pairs**

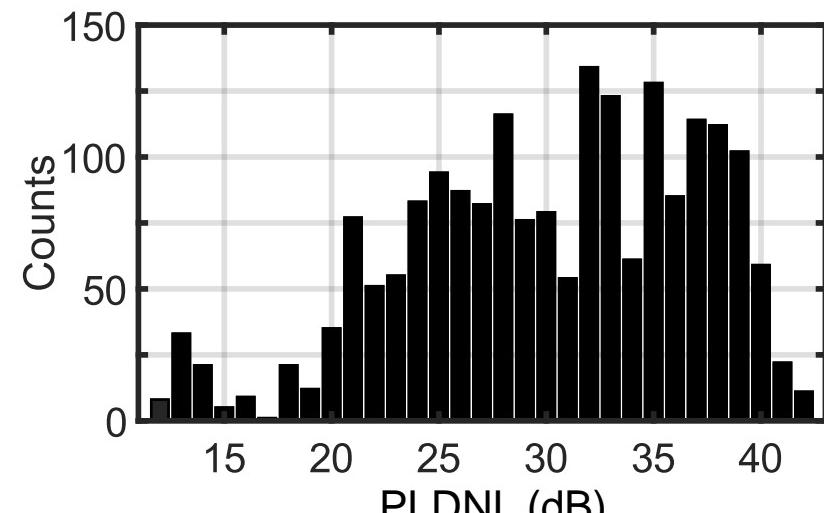


# QSF18 Dose-Response Data Summary



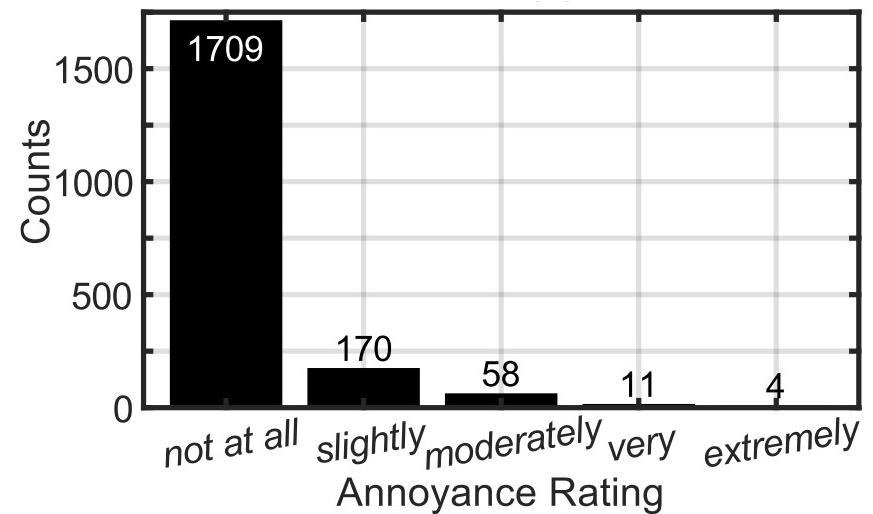
## ➤ Cumulative Dose

- 1,952 cumulative doses calculated from 8,704 single events
- Cumulative range in PLDNL: 7.3 to 41.1 dB



## ➤ Cumulative Response

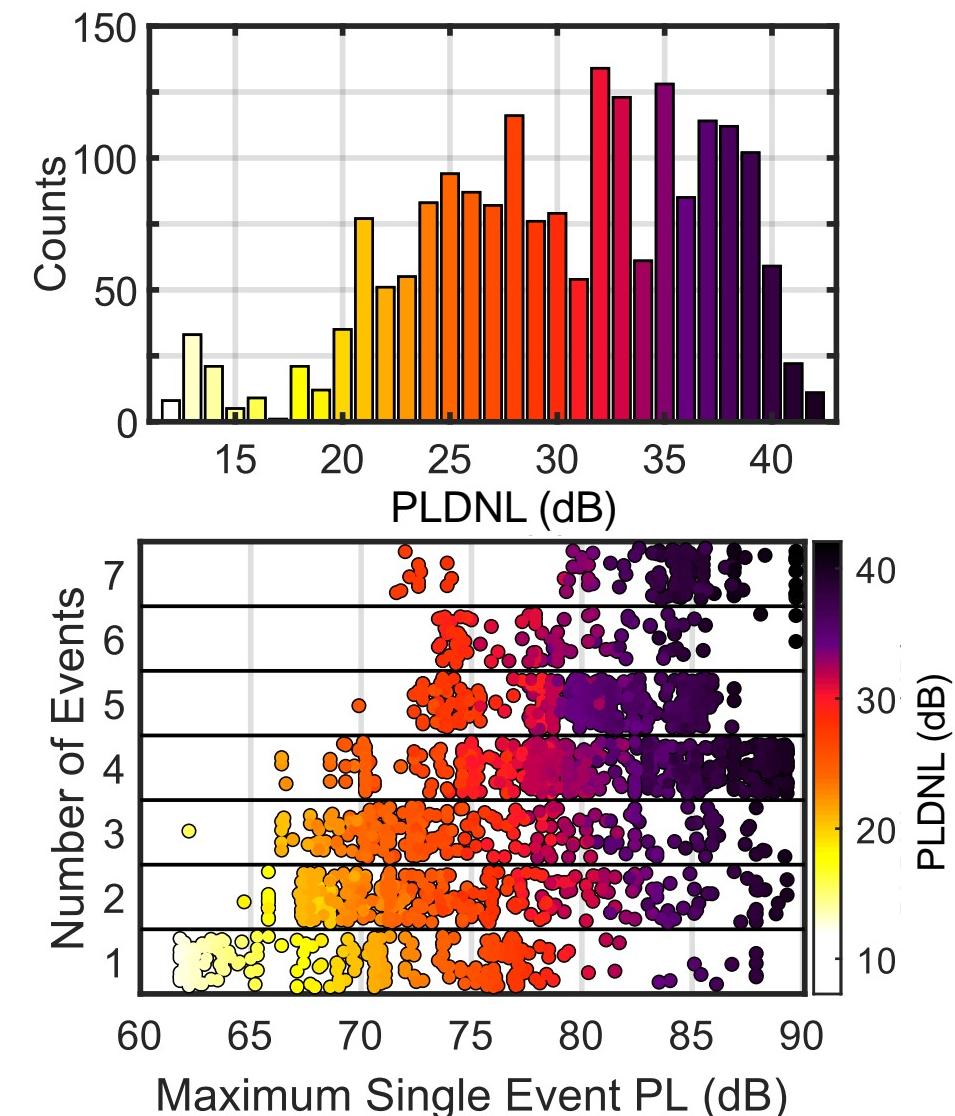
- 5-point verbal response scale
- Few annoyed responses
  - 15 (<1%) highly annoyed (HA)
  - 73 (4%) at least moderately annoyed (MA+)
  - 243 (12%) at least slightly annoyed (SA+)



# QSF18 Data: Max Single Event Level vs Number of Events



- Maximum Single Event PL vs Number of Events plot with DNL noted in color describes tested  $\Sigma$  space
- Maximum Single Event PL:
  - Ranges from 63 to 90 dB
- Number of Events:
  - 4 to 8 planned events per day (1 to 7 measured)
  - Doses only assigned when certain conditions met
- Multiple ways to achieve the same DNL
  - Trend dominated by maximum single event level due to logarithmic relationship

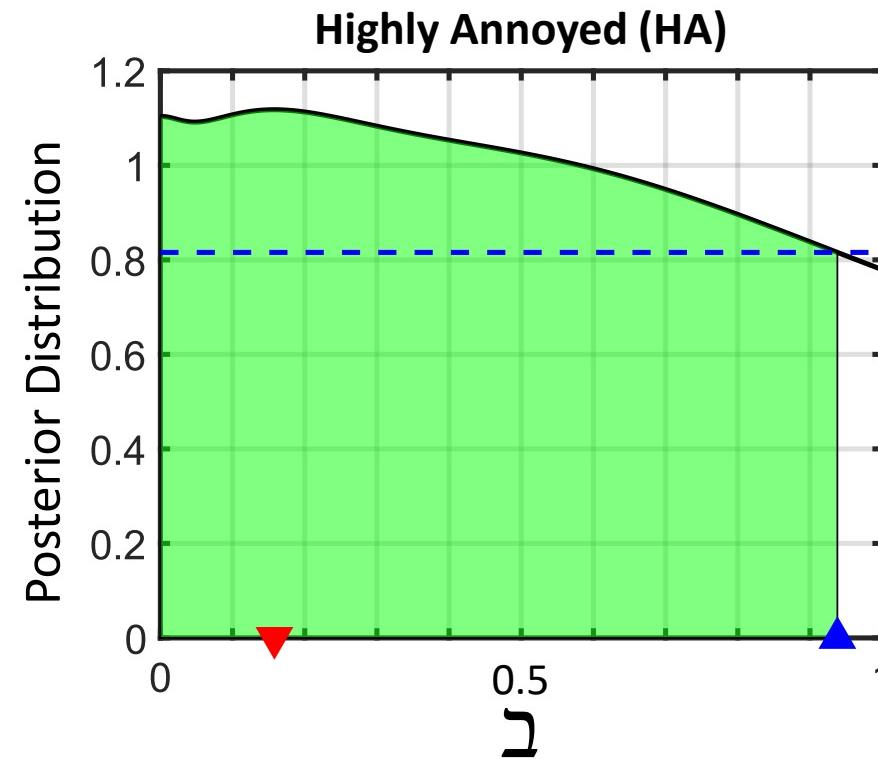
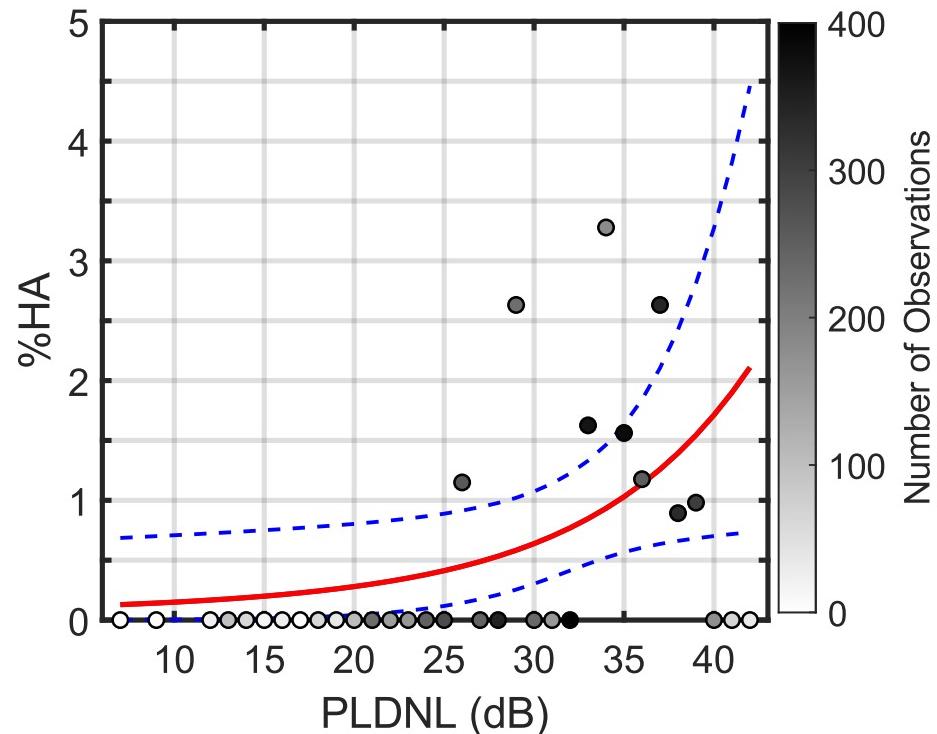


# QSF18 נ Posterior Distribution Results for HA



- Where is the peak?
- How narrow/broad is the peak?
- What  $\gamma$  values can be rejected?

- $\gamma \approx 0.16$  for HA
- Very broad peak; **not precise**
- $\gamma = 1$  is **weakly rejected**

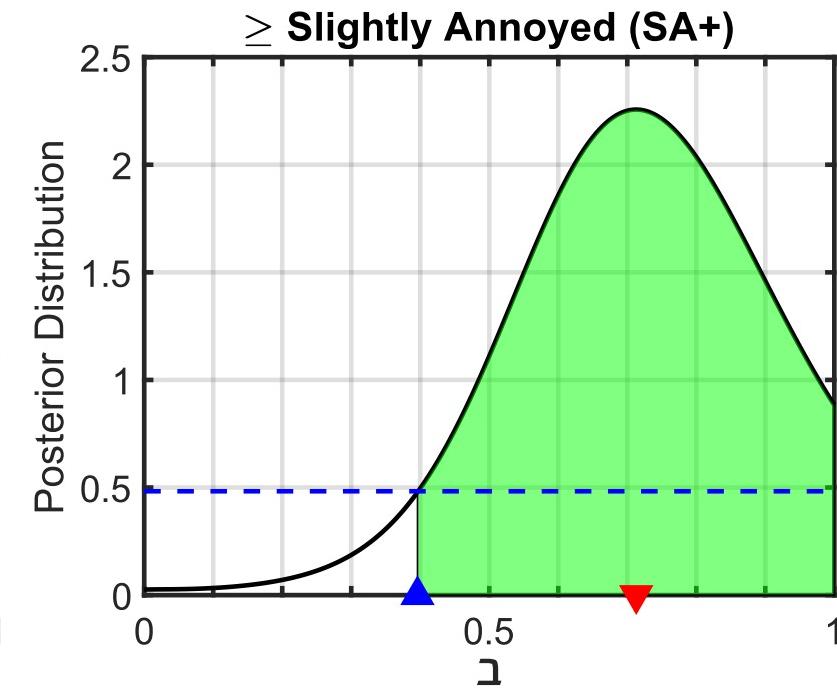
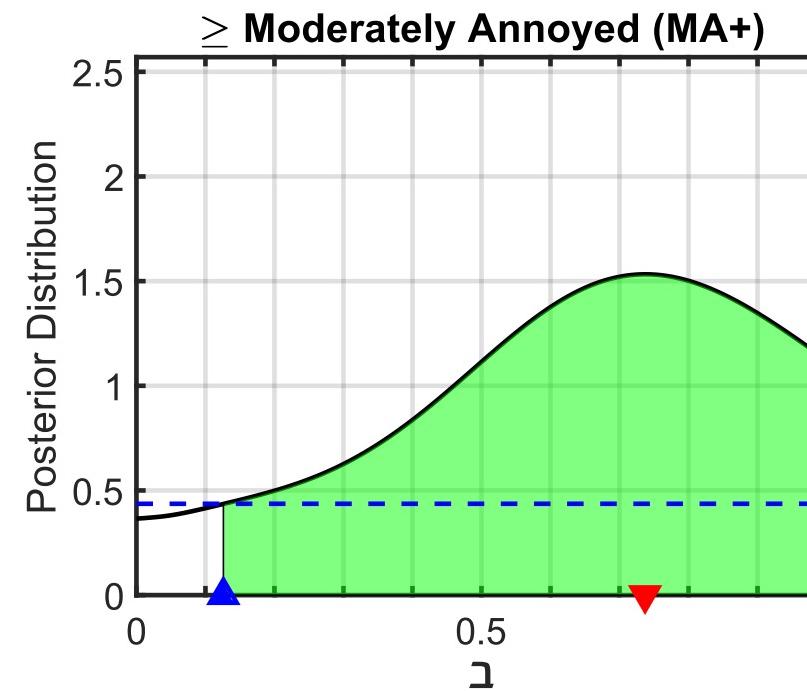
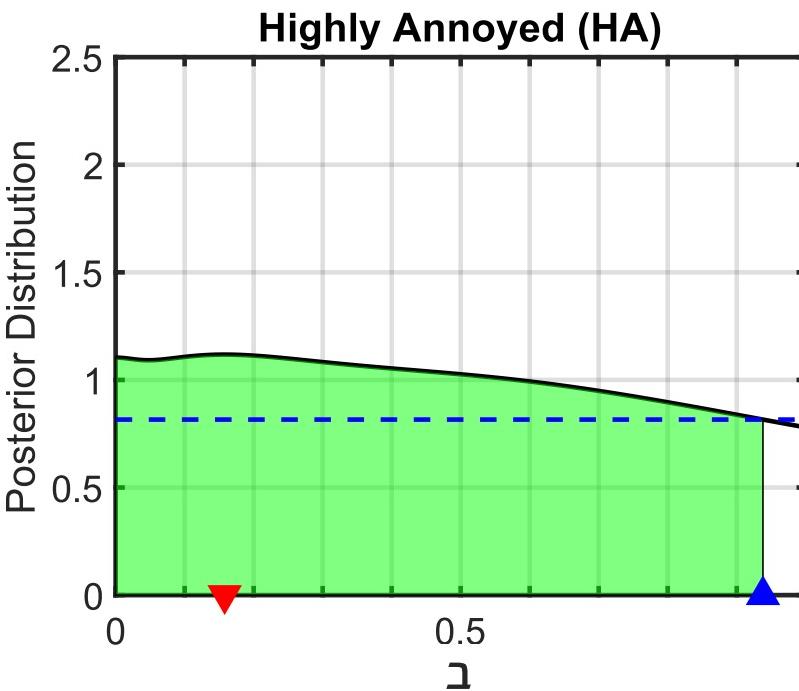


# QSF18 ↗ Posterior Distribution Results for HA, MA+, & SA+



- Where is the peak?
  - How narrow/broad is the peak?
  - What  $\gamma$  values can be rejected?
- $\gamma \approx 0.16$  for HA;  $\gamma \approx 0.72$  for MA+ & SA+
  - Very broad peak; **not precise**
  - $\gamma = 0$  is **rejected by MA+ & HA+**

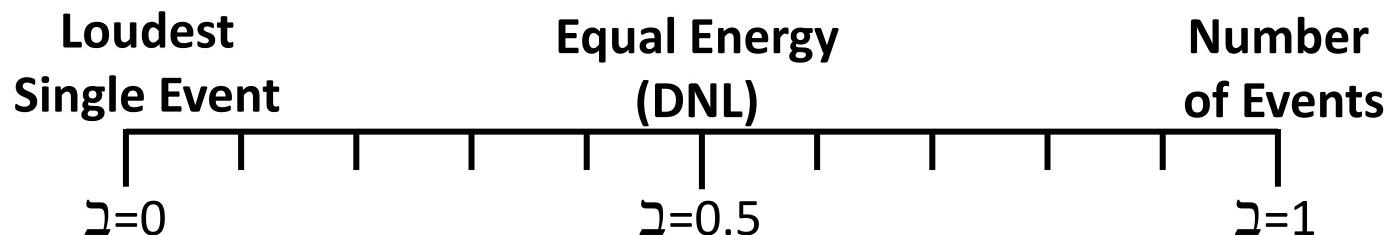
**People who are differently annoyed may be annoyed differently**



# Conclusion



- $\Sigma$  (“bet”) analysis works and provides insight into cumulative dose-response relationship



- Test design is important
  - Simulation demonstrates  $\Sigma$  analysis
  - QSF18 results are limited
  - Framework for future analysis of X-59 community data
- $\Sigma$  analysis is applicable to other noise studies
- Manuscript on  $\Sigma$  analysis in progress

